



Screening Corrections in the Spin Zero Approximation

Eman, B.

Publication date:
1966

Document Version
Publisher's PDF, also known as Version of record

[Link back to DTU Orbit](#)

Citation (APA):
Eman, B. (1966). *Screening Corrections in the Spin Zero Approximation*. Risø National Laboratory. Denmark. Forskningscenter Risoe. Risoe-R No. 146

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

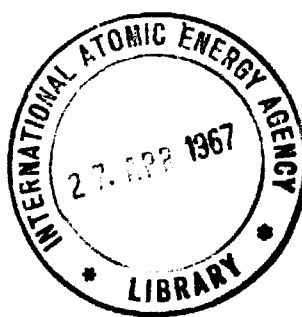
- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Danish Atomic Energy Commission
Research Establishment Risø

Screening Corrections in the Spin Zero Approximation

by **B. Eman**



November, 1966

Sales distributors: Jul. Gjellerup, 87, Søstergade, Copenhagen K, Denmark

Available on exchange from: Library, Danish Atomic Energy Commission, Risø, Roskilde, Denmark

November, 1966

Risø Report No. 146

Screening Corrections in the Spin Zero Approximation

by

B. Eman^x

The Danish Atomic Energy Commission
Research Establishment Risø
Physics Department

Abstract

Corrections for screening in beta decay are calculated from the solutions of the Klein-Gordon equations with Hulthén and Coulomb potentials. The parameters of the Hulthén potentials are obtained by fitting to the Hartree curves and the Fermi-Thomas-Dirac potentials respectively.

^x Permanent address: Institute "Ruder Boskovic", Zagreb, Yugoslavia

Contents

	Page
Introduction	3
1. Choice of the Potential	3
2. Results and Conclusion	4
Acknowledgements	5
Appendix	6
A. Klein-Gordon Equation with the Coulomb Potential	6
B. Klein-Gordon Equation with the Hulthén Potential	7
References	12
Tables and figure	13

Introduction

The Coulomb interaction between charged particles, created in weak processes, and the rest of the nucleus is modified by the charge distribution in the nucleus and by the presence of the atomic charge. These corrections, called finite-size and screening corrections respectively, have been analysed by different authors¹⁻⁶⁾. Finite-size corrections have been extensively calculated and tabulated^{4, 5)}. There is some ambiguity in the estimate of screening corrections since the choice of the screened potential affects the result. This potential is usually written in the form

$$V = \frac{Z a}{r} \phi(r) . \quad (1)$$

For $\phi(r)$, Reitz²⁾ used the solution of the Fermi-Thomas-Dirac (FTD) equation and Bühring⁶⁾ the approximate analytical expressions⁷⁾ of Hartree curves. In both cases the numerical integration of Dirac's equations was performed. The screening correction factors (ratio between Fermi functions for screened and Coulomb potentials) calculated by Reitz and by Bühring differ considerably.

We have introduced the assumption that the effect of screening on the radial wave functions of electrons is spin independent, i. e. it is the same whether the particle has spin one half or zero. The calculation of screening correction factors may then be considerably simplified because, as such a factor, we use the ratio between Fermi functions obtained from solutions of the Klein-Gordon equations with Hulthén⁸⁾ and Coulomb potentials respectively.

We first describe the determination of the parameters of the Hulthén potentials and then make a comparison with the results of Bühring. The method of solving the Klein-Gordon equation is outlined in the appendix.

1. Choice of the Potential

The solutions of the Klein-Gordon equation with zero angular momentum for the Hulthén potential

$$V_H = Z \cdot a \cdot \frac{\lambda e^{-\lambda r}}{1 - e^{-\lambda r}} \quad (2)$$

are available (see appendix and ref. 8). The parameter λ in (2) has to be chosen in such a way that eq. (2) reproduces the actual atomic potentials. It is obvious that with the one-parameter function (2) we shall be unable to reproduce the actual atomic potentials in a wide region of radius r . It seems reasonable to confine oneself to the region of small r because we make use of the solutions of the Klein-Gordon equation at the nuclear radius.

As an actual atomic potential we may use the values obtained by the Hartree self-consistent field calculations^{13,14)}. The Herman-Skillman tables of atomic potentials¹⁴⁾ differ from others¹³⁾ by taking into account that the electron does not interact with itself. Normalization of eq. (2) to the tabulated potentials¹⁴⁾ was done at $x = 0.01$, where x is the radius in Thomas-Fermi units, $x = \mu r = 1.13 \cdot a \cdot Z^{1/3} \cdot r$. The results obtained are given in fig. 1. Instead of plotting the parameter λ versus Z , we have plotted D , where $\lambda = 2 \cdot D \cdot a \cdot Z^{1/3}$.

On the other hand, when we use the tables of effective charges from ref. 13 and perform a normalization at the lowest tabulated value of the radius, the values obtained for the parameter D range from 1.2 (for $Z = 8$) to 1.55 (for $Z = 92$). The average value of D for $20 < Z < 40$ is 1.45.

As already pointed out, it is not possible to reproduce the tabulated potentials with the one-parameter function (2) in the wide region of radius r . So, for the sake of comparison we also give the values of the parameter D obtained when the normalization points were taken at large distances. The range of D is in this case between 1.10 (for $Z = 8$) and 0.60 (for $Z = 92$).

Of course, in comparison with the Hartree method, the Thomas-Fermi-Dirac (TFD) statistical model cannot be considered better for describing the actual atomic potential, but for the sake of completeness we calculated the parameters D by means of the TFD functions⁹⁾. At the nuclear radius D is 1.89 - 1.77, and for large distances, $0.4 \leq x \leq 1$, $D = 1.13$.

2. Results and Conclusion

The screening correction factors

$$S_{\text{scr}} = \frac{F_H(Z, E)}{F_C(Z, E)} \quad (3)$$

where

$$F_H(Z, E) = (\lambda R)^{2\sigma-2} \frac{\Gamma(\sigma+i\nu)\Gamma(\sigma+i\xi)}{\Gamma(2\sigma)\Gamma(1+2i\frac{p}{\lambda})} \left| \frac{F(\sigma+i\nu, \sigma+i\xi, 2\sigma, 1-e^{-\lambda R})}{\lambda} \right|^2 \quad (4)$$

and

$$F_c(Z, E) = (2pR)^{2\sigma-2} e^{\pi y} \left| \frac{\Gamma(\sigma+iy)}{\Gamma(2\sigma)} \right|^2 \left| {}_1F_1(\sigma+iy, 2\sigma, 2ipR) \right|^2 \quad (5)$$

(if $1-4\sigma^2 Z^2 > 0$; otherwise use equations (A 28) and (A 11)),

are calculated by using different values for the parameter D (tables 1 and 2), where $\lambda = 2 \cdot D \cdot \alpha \cdot Z^{1/3}$ and $R = 1.2 A^{1/3} \cdot 10^{-13}$ cm. For other notations in eqs. (4) and (5) see the appendix. In calculating the screening correction factor (3), only the first two terms in the expansion of hypergeometric functions are taken into account. We stress that the advantage gained by this method in terms of simplicity is paid for by the ad hoc nature of the assumption that the effect of screening is independent of the spin.

The format of our table 1 has been selected so as to allow easy comparison with similar tables of Bühring. The range of the parameter D in table 1 was taken so as to cover the possible values of D according to different models of atomic potentials. This allows us to estimate the error in actual calculations of screening corrections. The most probable values of parameter D are given in fig. 1. The results presented in tables 1 and 2 indicate that the corrections for screening in beta decay are smaller than was believed according to the results of Reitz, which are incorporated in the tables of Dzelepov and Zyrianova⁴⁾.

The measurement of the capture to beta plus ratios in heavy elements would be a good experimental check of the screening corrections. If we take the Reitz values of the screening correction factors, the K/β^+ ratios in the 800 keV transition of ¹⁷⁸Ta and the 790 keV transition of ²³⁴Np will be reduced by 14% and 17% respectively as compared with the values without any screening corrections. On the other hand, if we use our screening correction factors, these reductions will be only 6% in accordance with the results of Bühring.

Acknowledgements

It is a pleasure to thank Professor O. Kofoed-Hansen for his encouragement, Professor P. G. Hansen for helpful discussions, Leif Hansson of the GIER computer group at Risø for his kind assistance in the calculations, and the Danish Atomic Energy Commission for their hospitality at the Research Establishment Risø.

Appendix

A. Klein-Gordon Equation with the Coulomb Potential

The radial part of the Klein-Gordon equation for zero angular momentum

$$\left\{ \frac{d^2}{dr^2} + p - 2V(r)E + V^2(r) \right\} g_c(r) = 0 \quad (\text{A } 1)$$

and the Coulomb potential

$$V(r) = - \frac{Z \alpha}{r} \quad (\text{A } 2)$$

is of the type

$$\left\{ \frac{d^2}{dr^2} + A + \frac{B}{r} + \frac{C}{r^2} \right\} G(r) = 0. \quad (\text{A } 3)$$

The regular solution of equation (A 3) is¹¹⁾

$$G(r) = \frac{1}{2} \left| \frac{e^{i \frac{\pi}{2} k}}{\Gamma(2m+1)} \right| e^{-i \frac{\pi}{2} (m+1/2)} M_{k,m}(z), \quad (\text{A } 4)$$

where

$$M_{k,m}(z) = z^{1/2+m} e^{-z/2} {}_1F_1(1/2+m-k, 2m+1, z), \quad (\text{A } 5)$$

$$k = -i \frac{B}{2\sqrt{A}}, \quad m = \frac{1}{2} \sqrt{1-4C}, \quad z = 2i \sqrt{A} \cdot r$$

and ${}_1F_1(a, b, c)$ is a hypergeometric function.

According to (A 3), (A 4) and (A 5) the regular solution of equation (A 1) for the potential (A 2) is

$$g_c(r) = \frac{1}{2} e^{\frac{\pi y}{2}} \left| \frac{\Gamma(\sigma+iy)}{\Gamma(2\sigma)} \right| e^{-i \frac{\pi}{2} \sigma} (2ipr)^\sigma e^{-ipr} {}_1F_1(\sigma+iy, 2\sigma, 2ipr), \quad (\text{A } 6)$$

where

$$y = \frac{Z \alpha E}{p} \quad \text{and} \quad \sigma = \frac{1}{2} + \frac{1}{2} \sqrt{1 - 4 \alpha^2 Z^2}. \quad (\text{A } 7)$$

Because of the singularity of the function $g_c(r)$ the Fermi function $F_c(Z, E)$ has to be taken at the nuclear radius R :

$$F_c(Z, E) = \left| \frac{g_c(r)}{p \cdot r} \right|^2_{r=R} \quad (A 8)$$

For the sake of convenience we will change the notation slightly, putting

$$\sigma = \frac{1}{2} + S \quad (A 9)$$

$$S = \frac{1}{2} \left| \sqrt{1 - 4 a^2 Z^2} \right| .$$

Then

$$F_c(Z, E) = (2pR)^{2S-1} e^{\pi y} \left| \frac{\Gamma(\frac{1}{2} + S + iy)}{\Gamma(2S+1)} \right|^2 \left| {}_1F_1\left(\frac{1}{2} + S + iy, 1 + 2S, 2ipR\right) \right|^2 \quad (A 10)$$

for $1 - 4 a^2 Z^2 > 0$

and

$$F_c(Z, E) = \frac{e^{\pi(y-S)}}{2pR} \left| \frac{\Gamma(\frac{1}{2} + i(S+y))}{\Gamma(1+2iS)} \right|^2 \left| {}_1F_1\left(\frac{1}{2} + i(S+y), 1 + 2iS, 2ipR\right) \right|^2 \quad (A 11)$$

$$\text{for } 1 - 4 a^2 Z^2 < 0 .$$

B. Klein-Gordon Equation with the Hulthen Potential

Inserting in the Klein-Gordon equation (A 1) the Hulthén potential

$$V(r) = -Z a \frac{\lambda e^{-\lambda r}}{1 - e^{-\lambda r}} , \quad (A 2')$$

we obtain⁸⁾

$$\left\{ \frac{d^2}{dr^2} + p^2 + \frac{b e^{-\lambda r}}{1 - e^{-\lambda r}} + \frac{a e^{-2\lambda r}}{(1 - e^{-\lambda r})^2} \right\} g_H(r) = 0 , \quad (A 12)$$

where

$$a = a^2 \lambda^2 Z^2 \quad (A 13)$$

$$b = 2a \lambda Z E .$$

Inserting

$$g_H(r) = e^{-ipr} \phi(r) \quad (A 14)$$

and

$$t = e^{-\lambda r} \quad (A 15)$$

in eq. (A 2), we obtain

$$\left\{ \frac{d^2}{dt^2} + \frac{1+2i\frac{p}{\lambda}}{t} \frac{d}{dt} + \frac{b}{\lambda^2 t(1-t)} + \frac{a}{\lambda^2 (1-t)^2} \right\} \varphi(t) = 0. \quad (A 16)$$

In the next step the following substitution is useful:

$$\varphi(t) = (1-t)^\sigma \theta(t), \quad (A 17)$$

where σ is given by eq. (A 7). By inserting (A 17) into (A 16) we obtain

$$\begin{aligned} & \left\{ t(1-t) \frac{d^2}{dt^2} + \left[1+2i\frac{p}{\lambda} - (2\sigma+1+2i\frac{p}{\lambda})t \right] \frac{d}{dt} - \right. \\ & \left. - \left[\sigma(1+2i\frac{p}{\lambda}) - \frac{b}{\lambda^2} \right] \right\} \theta(t) = 0. \end{aligned} \quad (A 18)$$

In this way equation (A 12) is reduced to the hypergeometric equation

$$z(1-z) \frac{d^2 u}{dz^2} + [c - (a+b+1)z] \frac{du}{dz} - abu = 0. \quad (A 19)$$

Of 24 solutions of (A 19) (ref. 12, sections 14.3 and 14.4) we are interested in those with the boundary conditions $g_H(0) = 0$ and $\lim_{r \rightarrow \infty} g_H(r) = \sin(pr + \frac{\pi}{2})$. The solution

$$u_{17} = F(a, b, 1 - c + a + b, 1 - z) \quad (A 20)$$

satisfies these requirements.

From (A 20), (A 19), (A 18), (A 17), (A 16), (A 15), and (A 14) the non-normalized solution with the proper boundary conditions is

$$g_H(r) = N e^{-ipr} (1 - e^{-\lambda r}) F(\sigma + i\nu, \sigma + i\xi, 2\sigma, 1 - e^{-\lambda r}), \quad (A 21)$$

where

$$\nu = \frac{1}{\lambda} (p + \sqrt{p^2 + a - b}) \quad (A 22)$$

$$\xi = \frac{1}{\lambda} (p - \sqrt{p^2 + a - b}). \quad (A 23)$$

The normalization factor N can be determined by means of the relation between hypergeometric functions with z and $1-z$ (ref. 12, section 14.53)

$$\begin{aligned} \lim_{r \rightarrow \infty} e^{-ipr} F(\sigma+iv, \sigma+i\xi, 2\sigma, 1-e^{-\lambda r}) = \\ \lim_{r \rightarrow \infty} \left\{ \frac{\Gamma(1+2i\frac{p}{\lambda}) \Gamma(2\sigma)}{2i\frac{p}{\lambda} \Gamma(\sigma+iv) \Gamma(\sigma+i\xi)} e^{ipr} F(\sigma-iv, \sigma-i\xi, \right. \\ \left. 1-2i\frac{p}{\lambda}, e^{-\lambda r}) - \frac{\Gamma(1-2i\frac{p}{\lambda}) \Gamma(2\sigma)}{2i\frac{p}{\lambda} \Gamma(\sigma-iv) \Gamma(\sigma-i\xi)} e^{-ipr} \right. \\ \left. F(\sigma+iv, \sigma+i\xi, 1+2i\frac{p}{\lambda}, e^{-\lambda r}) \right\} = \\ \frac{\lambda}{p} \left| \frac{\Gamma(1+2i\frac{p}{\lambda}) \Gamma(2\sigma)}{\Gamma(\sigma+iv) \Gamma(\sigma+i\xi)} \right| \sin(pr + \Phi_0), \end{aligned} \quad (A 24)$$

where

$$\Phi_0 = \arg \left| \frac{\Gamma(2\sigma) \Gamma(1+2i\frac{p}{\lambda})}{\Gamma(\sigma+iv) \Gamma(\sigma+i\xi)} \right|.$$

From (A 24) we obtain for the normalization factor N

$$N = \frac{p}{\lambda} \left| \frac{\Gamma(\sigma+iv) \Gamma(\sigma+i\xi)}{\Gamma(2\sigma) \Gamma(1+2i\frac{p}{\lambda})} \right|. \quad (A 25)$$

Again the Fermi function has to be determined at the nuclear radius R:

$$F_H(Z, E) = \left| \frac{g_H(r)}{pr} \right|_{r=R}^2. \quad (A 26)$$

Using (A 25), (A 21) and (A 9), we have

$$F_H(Z, E) = (\lambda R)^{2S-1} \left| \frac{\Gamma(\frac{1}{2} + S + i\nu) \Gamma(\frac{1}{2} + S + i\xi)}{\Gamma(1+2S) \Gamma(1+2i\frac{p}{\lambda})} \right|^2 x \quad (A 27)$$

$$|F(\frac{1}{2} + S + i\nu, \frac{1}{2} + S + i\xi, 1 + 2S, 1 - e^{-\lambda R})|^2$$

$$\text{for } 1 - 4 a^2 Z^2 > 0$$

and

$$F_H(Z, E) = \frac{(1 - e^{-\lambda R})}{\lambda^2 R^2} \left| \frac{\Gamma(\frac{1}{2} + i(S + \nu)) \Gamma(\frac{1}{2} + i(S + \xi))}{\Gamma(1 + 2iS) \Gamma(1 + 2i\frac{p}{\lambda})} \right|^2 x \quad (A 28)$$

$$|F(\frac{1}{2} + i(S + \nu), \frac{1}{2} + i(S + \xi), 1 + 2iS, 1 - e^{-\lambda R})|^2$$

$$\text{for } 1 - 4 a^2 Z^2 < 0 .$$

We can also prove that our solutions $F_H(Z, E)$ approach $F_C(Z, E)$ when $\lambda \rightarrow 0$. From the definition of the hypergeometric functions it follows immediately that

$$\lim_{\lambda \rightarrow 0} F(\sigma + i\nu, \sigma + i\xi, 2\sigma, 1 - e^{-\lambda R}) = {}_1F_1(\sigma + iy, 2\sigma, 2ipR) . \quad (A 29)$$

For $|\Gamma(z)|^2$ the next approximation is valid; if $x \approx 1$ and $y \gg 1$, we have

$$|\Gamma(x + iy)|^2 \approx 2\pi y^{2x-1} e^{-\pi y} . \quad (A 30)$$

By way of example we shall calculate the limits of eq. (A 27). Using (A 29) and (A 30), we have

$$\lim_{\lambda \rightarrow 0} F_H(Z, E) = \lim_{\lambda \rightarrow 0} \left\{ (\lambda R)^{2S-1} \frac{v^{2S} e^{-\pi v}}{\frac{2p}{\lambda} e^{-\pi 2\frac{p}{\lambda}}} \left| \frac{\Gamma(\frac{1}{2} + S + i\xi)}{\Gamma(2S+1)} \right|^2 x \right. \\ \left. |F(\frac{1}{2} + S + i\nu, \frac{1}{2} + S + i\xi, 2S+1, 1 - e^{-\lambda R})|^2 \right\} .$$

As for $\lambda \rightarrow 0$

$$v \rightarrow \frac{2p}{\lambda} - \frac{Z\alpha E}{p}$$

$$\xi \rightarrow \frac{Z\alpha E}{p} = y ,$$

we have

$$\lim_{\lambda \rightarrow 0} F_H(Z, E) = (2pR)^{2S-1} e^{\pi y} \left| \frac{\Gamma(\frac{1}{2} + S + iy)}{\Gamma(2S + 1)} \right|^2 x$$

$$\left| {}_1F_1 \left(\frac{1}{2} + S + iy, 2S + 1, 2ipR \right) \right|^2 = F_c(Z, E) .$$

Q. E. D.

For positrons change Z to $-Z$ in all expressions.

References

- 1) M.E. Rose, Phys. Rev. 49 (1936) 727.
- 2) J.R. Reitz, Phys. Rev. 77 (1950) 10.
- 3) I.M. Band, L.N. Zyrianova and Tsin Chen-Zhui, Bulletin of the Academy of Science of the USSR, Phys. series 20 (1956) 1387; P.F. Zweifel, Phys. Rev. 96 (1954) 1572; Phys. Rev. 107 (1957) 329.
- 4) B.S. Dzelepov and L.N. Zyrianova, "Vlianie elektriceskovo polja atoma na beta-raspad" (Akademi Nauk SSSR, Moscow, 1956).
- 5) C.P. Bhalla and M.E. Rose, ORNL-3207 (1962).
- 6) W. Bühring, Nuclear Physics 61 (1965) 110.
- 7) W.J. Byatt, Phys. Rev. 104 (1956) 1298.
- 8) L. Durand III, Phys. Rev. 135 (1964) B 310.
- 9) R.P. Feynman, N. Metropolis and E. Teller, Phys. Rev. 75 (1949) 1561.
- 10) N.F. Mott and H.S.W. Massay, "The Theory of Atomic Collisions" (Clarendon, Oxford, 1965) 460-462.
- 11) M.H. Hull and G. Breit, Handbuch der Physik XLI/1 (Springer-Verlag, Berlin, 1959) 408.
F.L. Yost, J.A. Wheeler and G. Breit, Phys. Rev. 49 (1936) 174.
- 12) E.T. Whittaker and G.N. Watson, "Modern Analysis" (Cambridge University Press, 1952).
- 13) A.J. Freeman, Phys. Rev. 91 (1953) 1410. For other references see H.H. Landolt and R. Börnstein, "Zahlenwerte und Funktionen aus Physik, Chemie, Astronomie Geophysik und Technik", Springer-Verlag, Berlin, 1950, sixth edition, vol. I, page 276.
- 14) F. Herman and S. Skillman, "Atomic Structure Calculations" (Prentice-Hall Inc., New Jersey, 1963).

Table 1

Screening correction factors S_{scr}^{κ}

Z(A)	17(35)			30(64)			50(114)			7(14)			-26(54)			-58(136)					
	D	1.45	0.76	B	D	1.45	0.70	B	D	1.45	0.66	B	D	1.45	1.13	0.66	B	D	1.45	1.13	0.66
0.2	0.9923	0.9923	0.9959	0.9943	0.9963	0.9991	0.9911	0.9911	0.9977	0.9977	0.9977	0.9977	1.0449	2.6243	1.7855	1.4311	1.9338	21.094	13.325	5.9485	16.870
0.4	0.9922	0.9958	0.9958	0.9938	0.9929	0.9966	0.9892	0.9892	0.9978	0.9978	0.9978	0.9978	1.0093	1.1181	1.0923	1.0523	1.1334	1.7396	1.5655	1.3204	1.7836
0.6	0.9945	0.9971	0.9971	0.9913	0.9930	0.9966	0.9901	0.9901	0.9974	0.9974	0.9974	0.9974	1.0043	1.0406	1.0317	1.0186	1.0300	1.2033	1.1866	1.0927	1.2390
0.8	0.9960	0.9978	0.9978	0.9934	0.9940	0.9971	0.9912	0.9912	0.9974	0.9974	0.9974	0.9974	1.0027	1.0207	1.0162	1.0095	1.034	1.0889	1.0694	1.0408	1.1158
1.0	0.9970	0.9984	0.9984	0.9941	0.9950	0.9976	0.9924	0.9924	0.9975	0.9975	0.9975	0.9975	1.0020	1.0130	1.0101	1.0059	1.070	1.0489	1.0322	1.0225	1.0704
1.2	0.9976	0.9988	0.9988	0.9953	0.9958	0.9980	0.9933	0.9933	0.9978	0.9978	0.9978	0.9978	1.0016	1.0093	1.0072	1.0042	1.0123	1.0310	1.0242	1.0142	1.0488
1.4	0.9981	0.9990	0.9990	0.9959	0.9964	0.9983	0.9940	0.9940	0.9980	0.9980	0.9980	0.9980	1.0013	1.0072	1.0056	1.0033	1.0102	1.0217	1.0170	1.0099	1.0370
1.6	0.9984	0.9992	0.9992	0.9961	0.9969	0.9985	0.9946	0.9946	0.9982	0.9982	0.9982	0.9982	1.0011	1.0058	1.0046	1.0027	1.0088	1.0163	1.0127	1.0075	1.0297
1.8	0.9987	0.9994	0.9994	0.9965	0.9973	0.9987	0.9953	0.9953	0.9984	0.9984	0.9984	0.9984	1.0010	1.0050	1.0039	1.0024	1.0075	1.0129	1.0101	1.0059	1.0249
2.0	0.9989	0.9995	0.9995	0.9967	0.9977	0.9988	0.9956	0.9956	0.9986	0.9986	0.9986	0.9986	1.0009	1.0043	1.0034	1.0021	1.0066	1.0107	1.0083	1.0049	1.0214

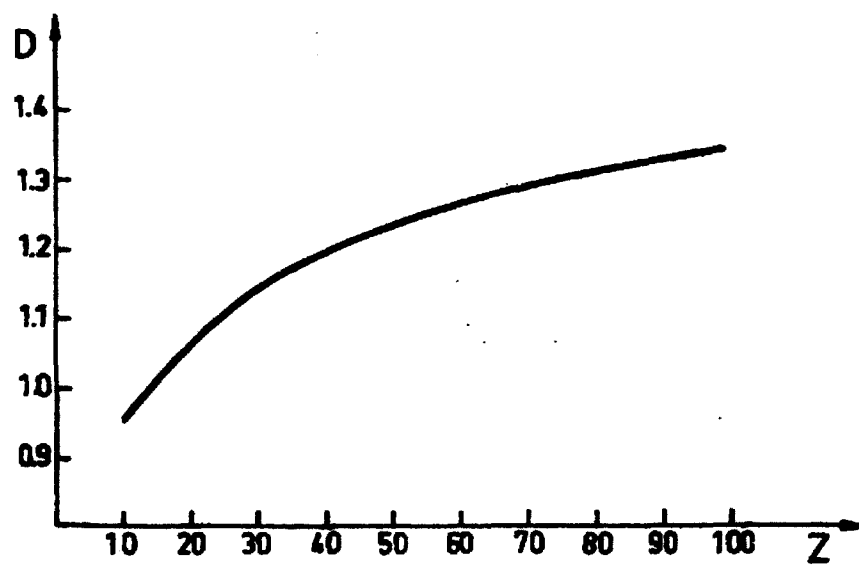
* Upper and lower values for the parameter D are obtained from Hartree potentials at small and large values of the radius, respectively. The value $D = 1.13$ corresponds to the TFD potential³⁾ and to the Fermi-Thomas radius $r \approx 0.4$.

† Mahring's results⁶⁾.

Table 2

Screening correction factors obtained by using the values of the parameter D from fig. 1

Z(A)	-20(40)	-30(60)	-40(81)	-50(119)	-60(144)	-70(173)	-80(200)	-90(232)
0.2	1.3574	2.1977	4.1269	9.2729	22.802	68.747	221.84	746.94
0.4	1.0491	1.1275	1.2580	1.4617	1.7649	2.2101	2.6920	3.6732
0.6	1.0179	1.0424	1.0789	1.1339	1.2067	1.2992	1.4276	1.5883
0.8	1.0095	1.0210	1.0376	1.0602	1.0890	1.1220	1.1697	1.2264
1.0	1.0062	1.0129	1.0231	1.0359	1.0482	1.0621	1.0750	1.1119
1.2	1.0045	1.0091	1.0149	1.0220	1.0301	1.0382	1.0468	1.0636
1.4	1.0036	1.0069	1.0110	1.0157	1.0207	1.0251	1.0306	1.0396
1.6	1.0030	1.0056	1.0087	1.0121	1.0153	1.0188	1.0229	1.0287
1.8	1.0025	1.0047	1.0071	1.0097	1.0119	1.0145	1.0169	1.0190
2.0	1.0022	1.0041	1.0061	1.0081	1.0097	1.0112	1.0121	1.0141



Parameter D of the Hulthen potential as a function of the atomic charge Z .